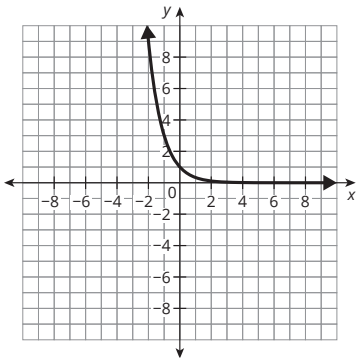
Notes

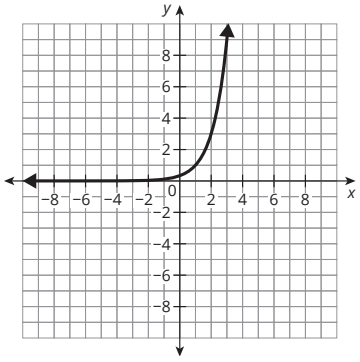
Name _____ D ate _____

1. Which graph best represents $f(x) = -3^x$?

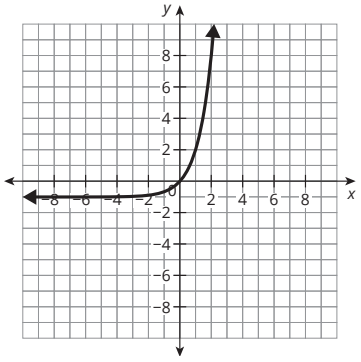
A.



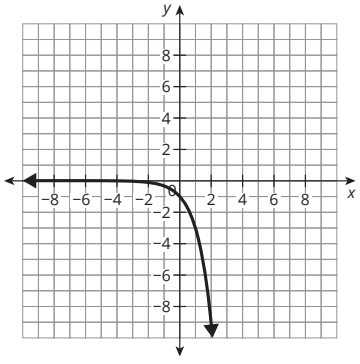
B.



C.



★ **D.**



For the equation, when $x = 0$, $f(x) = -1$. The point $(0, -1)$ must be on the graph.

MODULE 4 • TOPIC 1 • END OF TOPIC ASSESSMENT 37

Notes

2. Rewrite $\sqrt[3]{5^2}$ with a rational exponent.

F. $5^{\frac{3}{2}}$

G. $3^{\frac{2}{5}}$

★ H. $5^{\frac{2}{3}}$

J. $2^{\frac{5}{3}}$

The cube root of 5^2 is 5^2 raised to the $\frac{1}{3}$ power. To simplify a power raised to a power, multiply exponents.

3. Gabriel buys 512 bottles of water to hand to runners in a marathon. He estimates that each hour, half of the remaining bottles will be used. Which function can be used to determine the number of bottles remaining at the end of x hours?

A. $f(x) = 512(0.05)^x$

★ B. $f(x) = 512(0.50)^x$

C. $f(x) = 512(1.05)^x$

D. $f(x) = 512(1.50)^x$

The initial value (a-value) is 512. The constant ratio (b value) is 0.5.

4. Write an equivalent expression in simplest form.

$$\frac{90r^{-6}y^7z^{12}}{15r^{-9}y^{10}z^{-5}}$$

$$= \left(\frac{90}{15}\right)(r^{-6-(-9)})(y^{7-10})(z^{12-(-5)})$$

$$= 6r^3y^{-3}z^{17}$$

$$= \frac{6r^3z^{17}}{y^3}$$

38 MODULE 4 • TOPIC 1 • END OF TOPIC ASSESSMENT

Use the explicit formula to answer Questions 5 and 6.

$g(n) = -2\left(\frac{1}{5}\right)^{n-1}$

5. Rewrite the explicit formula in exponential function form, $f(x) = a \cdot b^x$.

$g(n) = -2 \cdot \left(\frac{1}{5}\right)^{n-1}$

$g(n) = -2\frac{\left(\frac{1}{5}\right)^{n}}{\left(\frac{1}{5}\right)^{1}}$

$g(n) = -2\left(\frac{1}{5}\right)^{n}(5)$

$g(n) = -10\left(\frac{1}{5}\right)^{n}$

6. Identify the y-intercept, the equation for the asymptote, and whether the function increases or decreases from left to right.

$g(n) = -10\left(\frac{1}{5}\right)^{n}$

$g(0) = -10\left(\frac{1}{5}\right)^{0}$

$g(0) = -10(1)$

$g(0) = -10$

y-intercept: -10

asymptote: $y = 0$

The function is increasing from left to right.

Write the expression in simplest radical form to answer Questions 7 and 8.

7. $\sqrt{32}$

$\sqrt{32}$

$= \sqrt{16 \cdot 2}$

$4\sqrt{2}$

8. $5\sqrt{117}$

$5\sqrt{117}$

$= 5\sqrt{9 \cdot 13}$

$= 5 \cdot 3\sqrt{13}$

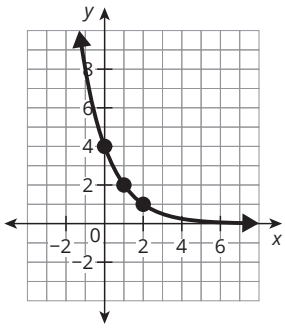
$15\sqrt{13}$

Notes

MODULE 4 • TOPIC 1 • END OF TOPIC ASSESSMENT 39

Notes

Use the graph of the exponential function to answer Questions 9 and 10.



9. Determine the domain and range of the function.

   Domain: $-\infty < x < \infty$

   Range: $0 < y < \infty$

10. Write an exponential function of the form $f(x) = ab^x$ for the graph.

    The y-intercept is 4, which is the a-value. The constant ratio is $\frac{1}{2}$, which is the b-value. The exponential function is $f(x) = 4\left(\frac{1}{2}\right)^x$.

40 MODULE 4 • TOPIC 1 • END OF TOPIC ASSESSMENT

Notes

Use the table to answer Questions 11 and 12.

The table shows the number of bacteria in a Petri dish over a three-hour period.

Bacteria	
Time (hours)	Number of Bacteria
0	460
1	1380
2	4140
3	12,420

11. Write an exponential function to represent the number of bacteria in the Petri dish as a function of time, t.

The value of y when $t = 0$ is 460, which is the a-value. The constant ratio is 3, which is the b-value. The exponential function is $f(t) = 460 \cdot 3^t$

12. Identify the y-intercept and the constant ratio of the exponential function and interpret them in terms of the scenario.

The y-intercept is 460, and this represents the initial number of bacteria. The constant ratio is 3, and it represents that the number of bacteria triples every hour.

MODULE 4 • TOPIC 1 • END OF TOPIC ASSESSMENT 41

Notes

13. Rewrite the expression in simplest radical form.
$(3r^{\frac{1}{3}}s^{\frac{1}{2}})(3rs^{\frac{1}{2}})$

$(3r^{\frac{1}{3}}s^{\frac{1}{2}})(3rs^{\frac{1}{2}}) = (3)(3)(r^{\frac{1}{3}+1})(s^{\frac{1}{2}+\frac{1}{2}})$

$9r^{\frac{4}{3}}s = 9s(\sqrt[3]{r})^4$

14. A sequence is defined by $f(n) = \frac{1}{2}f(n-1)$ for each whole number n, where $n > 1$. What are the first four terms of the sequence when $f(1) = 36$?

36, 18, 9, 4.5

42 MODULE 4 • TOPIC 1 • END OF TOPIC ASSESSMENT

MODULE 4, TOPIC 1 ASSESSMENT SCORING GUIDE

1 Introduction to Exponential Functions

Exponential Functions and Equations

1 DAY = 45-MINUTE SESSION

The student is expected to:

A.9A determine the domain and range of exponential functions of the form $f(x) = ab^x$ and represent the domain and range using inequalities.

A.9B interpret the meaning of the values of a and b in exponential functions of the form $f(x) = ab^x$ in real-world problems.

A.9C write exponential functions in the form $f(x) = ab^x$ (where b is a rational number) to describe problems arising from mathematical and real-world situations, including growth and decay.

A.9D graph exponential functions that model growth and decay and identify key features, including y-intercept and asymptote, in mathematical and real-world problems.

Number and Algebraic Methods

The student is expected to:

A.11A simplify numerical radical expressions involving square roots.

A.11B simplify numeric and algebraic expressions using the laws of exponents, including integral and rational exponents.

A.12C identify terms of arithmetic and geometric sequences when the sequences are given in function form using recursive processes.

Question Number	TEKS	Point Value	Scoring Guidance
1	**A.9D**	1	• The student selects the correct answer. (1 point) • The student does not select the correct answer. (0 points)
2	**A.11B**	1	• The student selects the correct answer. (1 point) • The student does not select the correct answer. (0 points)
3	**A.9C**	1	• The student selects the correct answer. (1 point) • The student does not select the correct answer. (0 points)
4	**A.11B**	1	• The student correctly simplifies the expression. (1 point) • The student does not correctly simplify the expression. (0 points)
5	**A.9C**	1	• The student correctly converts the formula. (1 point) • The student does not correctly convert the formula. (0 points)
6	**A.9D**	2	• The student correctly identifies all characteristics of the function. (2 points) • The student correctly identifies some characteristics of the function. (1 point) • The student does not correctly identify any characteristics of the function. (0 points)
7	A.11A	1	• The student correctly simplifies the expression. (1 point) • The student does not correctly simplify the expression. (0 points)
8	A.11A	1	• The student correctly simplifies the expression. (1 point) • The student does not correctly simplify the expression. (0 points)

*Bold TEKS = Readiness Standard

Question Number	TEKS*	Point Value	Scoring Guidance
9	A.9A	2	• The student determines the correct domain and range of the function. (2 points) • The student determines the correct domain or range of the function. (1 point) • The student does not determine the correct domain and range of the function. (0 points)
10	**A.9C**	1	• The student determines the correct function. (1 point) • The student does not determine the correct function. (0 points)
11	**A.9C**	1	• The student determines the correct function. (1 point) • The student does not determine the correct function. (0 points)
12	A.9B	2	• The student correctly identifies the characteristics and correctly interprets them. (2 points) • The student correctly identifies the characteristics or correctly interprets them. (1 point) • The student does not correctly identify the characteristics nor correctly interprets them. (0 points)
13	**A.11B**	1	• The student correctly simplifies the expression. (1 point) • The student does not correctly simplify the expression. (0 points)
14	A.12C	1	• The student correctly determines the first four terms of the sequence. (1 point) • The student does not correctly determine the first four terms of the sequence. (0 points)

*Bold TEKS = Readiness Standard

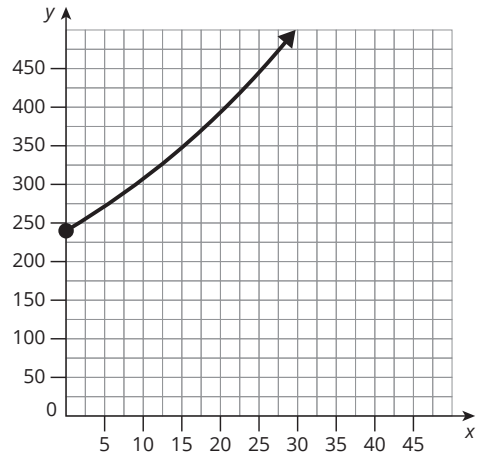
Response to Student Performance		
TEKS*	**Question(s)**	**Recommendations**
A.9A	9	**To support students:** • Review domain and range. • Use Skills Practice Set V.A for additional practice.
A.9B	12	**To support students:** • Use Skills Practice Set III.B for additional practice. • Review Lesson 3 Assignment Practice Question 2a.
A.9C	3, 9, 10, 11	**To support students:** • Use Skills Practice Set III.B for additional practice. • Review Lesson 3 Assignment Practice Questions 1 and 2.
	5	**To support students:** • Use Skills Practice Set III.A for additional practice. • Review Lesson 2 Assignment Practice Questions 5, 6, and 8.
A.9D	1, 6	**To support students:** • Use Skills Practice Set IV.A for additional practice. • Review Lesson 3 Assignment Practice Question 2. **To challenge students:** • Extend student knowledge with Skills Practice Extension Set III.
A.11A	7, 8	**To support students:** • Use Skills Practice Set IV.B for additional practice. • Review Lesson 4 Assignment Practice Questions 19 through 26.
A.11B	2	**To support students:** • Use Skills Practice Sets IV.B and IV.D for additional practice. • Review Lesson 4 Assignment Practice Questions 1 through 4 and 15 through 18. **To challenge students:** • Extend student knowledge with Skills Practice Extension Set IV.
	4	**To support students:** • Use Skills Practice Sets I.B, II.A, II.B, and II.C for additional practice. • Review Lesson 2 Assignment Practice Questions 1 through 8. **To challenge students:** • Extend student knowledge with Skills Practice Extension Sets I and II.
	13	**To support students:** • Use Skills Practice Sets IV.C and IV.E for additional practice. • Review Lesson 4 Assignment Practice Questions 5 through 14. **To challenge students:** • Extend student knowledge with Skills Practice Extension Set IV.
A.12C	14	**To support students:** • Review determining terms of a sequence. • Use Skills Practice Section IV. Spaced Practice for additional practice.
NOTE: Both teachers and administrators should refer to the Assessment Guidance and Analysis section of the Course and Implementation Guide for additional support in analyzing and responding to student data.		

*Bold TEKS = Readiness Standard

Notes

Name _____ D ate _____

1. Which scenario and equation is represented by the graph?



A. Carlos deposits $200 in a savings account with an interest rate of 2% simple interest.

$I = 200(0.02)t$

B. Carlos deposits $240 in a savings account with an interest rate of 2.5% simple interest.

$I = 240(0.25)t$

C. Carlos deposits $200 in a savings account with an interest rate of 2% compounded annually.

$P(t) = 200 \cdot (1.02)^t$

★ **D.** Carlos deposits $240 in a savings account with an interest rate of 2.5% compounded annually.

$P(t) = 240 \cdot (1.025)^t$

The *y*-intercept of the graph is about 240, so the *a*-value should be 240. Since compound interest has a rate of growth, the *b*-value should be greater than 1.

MODULE 4 • TOPIC 2 • END OF TOPIC ASSESSMENT 43

Notes

2. The function $f(x) = 125{,}000(1.028)^x$ models the change in population of a city, where x represents the number of years since 2010. Which statement is **NOT** true?

F. The domain of the function includes all real values of x.

G. The point (1, 128,500) represents the population of the city in 2011.

H. The population of the city in 2010 is 125,000.

★ **J.** The function has an asymptote at $y = 125{,}000$.

The asymptote of the function is $y = 0$.

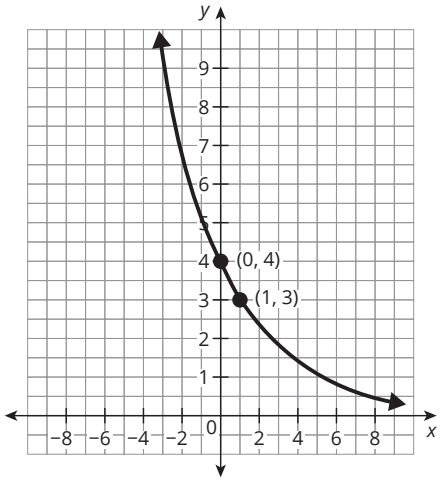
3. Write an exponential function that models the values in the table.

x	y
0	−1
1	−3
2	−9
3	−27

The value of $f(x)$ when $x = 0$ is (-1), and the constant ratio is 3.

$f(x) = -(3)^x$

4. Write an exponential function that models the graph.



The y-intercept is 4 and the constant ratio is $\frac{3}{4}$.

$f(x) = 4\left(\frac{3}{4}\right)^x$

44 MODULE 4 • TOPIC 2 • END OF TOPIC ASSESSMENT

Notes

5. A scientist puts a sample of 4000 bacteria in a Petri dish. The number of bacteria decrease at a rate of 2% per hour. Write an exponential function to represent the number of bacteria, b, as a function of time measured in hours, t.

   The initial number of bacteria is 4,000, and the decay rate is $1 - 0.02 = 0.98$.

   $b(t) = 4000 \cdot (0.98)^t$

Use the information and the table to answer Questions 6 through 9.

A concert venue opened in 2012. The table shows the total number of people at the concert venue the first few years it was open.

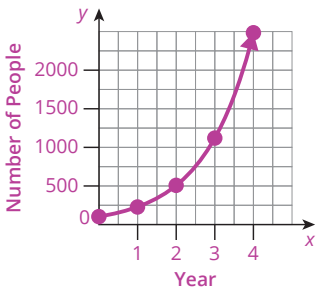
Year	2012	2013	2014	2015	2016
Number of People	103	224	516	1118	2487

6. Let x represent the number of years since 2012 and $f(x)$ represent the total number of people at the concert venue each year. Write an exponential function to best model the values in the table. Round all values to the hundredths place.

   Use technology to find the function.

   $f(x) = 102.61(2.22)^x$

7. Sketch the graph for the function you wrote in Question 6. Include the data points from the table on your graph.



8. In the exponential function form $f(x) = ab^x$, what is the meaning of a and b for this scenario?

   The a-value is the number of people in the year the venue opened, which is about 103. The b-value is the rate at which the number of people at the venue increased. The number of people increased by about 122% each year.

9. About how many people are predicted to be at the concert venue in 2017?

   2017 is 5 years after 2012.

   $f(5) = 102.61(2.22)^5$

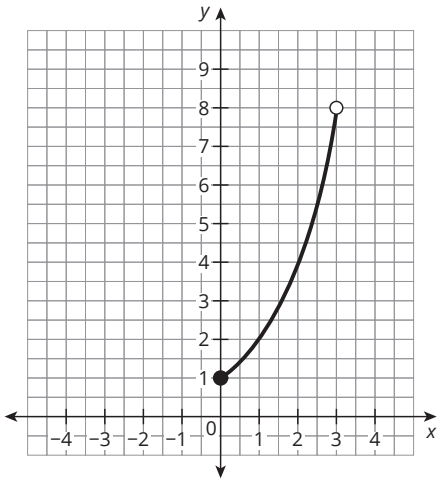
   $f(5) = 5532.92$

   There will be about 5533 people at the venue in 2017.

MODULE 4 • TOPIC 2 • END OF TOPIC ASSESSMENT 45

Notes

10. Write the domain and range of the partial exponential function using inequalities.



Domain: $0 \leq x < 3$

Range: $1 \leq y < 8$

11. Robert uses $f(x) = 2200(1.04)^x$ to calculate the interest he earns each year for his savings account. What is the yearly interest rate as a percent?

The b-value in the equation is the growth rate, which is 1 + the interest rate. The interest rate is 0.04, or 4%.

12. The expression $(x^{23})(x^{\frac{1}{4}})^8$ is equivalent to x^p. What is the value of p?

$(x^{23})(x^{\frac{1}{4}})^8$

$= (x^{23})(x^2)$

$= x^{25}$

$p = 25$

46 MODULE 4 • TOPIC 2 • END OF TOPIC ASSESSMENT

13. Isaiah deposited $1500 in an account that earns simple interest each year. The table shows the time in years since the money was deposited and the balance in the account.

Time (years)	Simple Interest Balance (dollars)
3	1612.50
5	1687.50
8	1800

What is the rate of change of the simple interest balance in dollars with respect to the time in years since the money was deposited?

$\frac{1800 - 1687.50}{8 - 5} = \frac{112.50}{3} = 37.50$

The rate of change is $37.50.

Notes

MODULE 4 • TOPIC 2 • END OF TOPIC ASSESSMENT 47

MODULE 4, TOPIC 2 ASSESSMENT SCORING GUIDE

2 Using Exponential Equations

Linear Functions, Equations, and Inequalities

1 DAY = 45-MINUTE SESSION

The student is expected to:

A.3B calculate the rate of change of a linear function represented tabularly, graphically, or algebraically in context of mathematical and real-world problems.

Exponential Functions and Equations

The student is expected to:

A.9A determine the domain and range of exponential functions of the form $f(x) = ab^x$ and represent the domain and range using inequalities.

A.9B interpret the meaning of the values of a and b in exponential functions of the form $f(x) = ab^x$ in real-world problems.

A.9C write exponential functions in the form $f(x) = ab^x$ (where b is a rational number) to describe problems arising from mathematical and real-world situations, including growth and decay.

A.9D graph exponential functions that model growth and decay and identify key features, including y-intercept and asymptote, in mathematical and real-world problems.

A.9E write, using technology, exponential functions that provide a reasonable fit to data and make predictions for real-world problems.

Number and Algebraic Methods

The student is expected to:

A.11B simplify numeric and algebraic expressions using the laws of exponents, including integral and rational exponents.

A.12B evaluate functions, expressed in function notation, given one or more elements in their domains.

Question Number	TEKS*	Point Value	Scoring Guidance
1	**A.9D**	1	• The student selects the correct answer. (1 point) • The student does not select the correct answer. (0 points)
2	**A.9D**	1	• The student selects the correct answer. (1 point) • The student does not select the correct answer. (0 points)
3	**A.9C**	1	• The student determines the correct function. (1 point) • The student does not determine the correct function. (0 points)
4	**A.9C**	1	• The student determines the correct function. (1 point) • The student does not determine the correct function. (0 points)
5	**A.9C**	1	• The student determines the correct function. (1 point) • The student does not determine the correct function. (0 points)
6	A.9E	1	• The student determines the correct function. (1 point) • The student does not determine the correct function. (0 points)
7	**A.9D**	1	• The student correctly graphs the function. (1 point) • The student does not correctly graph the function. (0 points)

*Bold TEKS = Readiness Standard

Question Number	TEKS*	Point Value	Scoring Guidance
8	A.9B	2	• The student correctly explains the meaning of the *a*-value and the *b*-value. (2 points) • The student correctly explains the meaning of the *a*-value or the *b*-value. (1 point) • The student does not correctly explain the meaning of the *a*-value nor the *b*-value. (0 points)
9	A.12B	1	• The student makes a reasonable prediction of the number of people. (1 point) • The student does not make a reasonable prediction of the number of people. (0 points)
10	A.9A	2	• The student writes correct inequalities to describe the domain and the range. (2 points) • The student writes a correct inequality to describe the domain or the range. (1 point) • The student does not write a correct inequality to describe the domain nor the range. (0 points)
11	A.9B	1	• The student correctly identifies the percent. (1 point) • The student does not correctly identify the percent. (0 points)
12	**A.11B**	1	• The student correctly determines the value of *p*. (1 point) • The student does not correctly determine the value of *p*. (0 points)
13	**A.3B**	1	• The student correctly determines the rate of change. (1 point) • The student does not correctly determine the rate of change. (0 points)

*Bold TEKS = Readiness Standard

Response to Student Performance		
TEKS*	**Question(s)**	**Recommendations**
A.3B	13	**To support students:** • Review Lesson 1 Assignment Practice Question 1. • Review Lesson 1, Activity 1.1, Question 3.
A.9A	10	**To support students:** • Use Skills Practice Sets III.B and III.D for additional practice. • Review Lesson 2 Assignment Practice Question 3.
A.9B	8, 11	**To support students:** • Use Skills Practice Sets II.D and III.D for additional practice. • Review Lesson 2 Assignment Practice Questions 2 and 3. **To challenge students:** • Extend student knowledge with Skills Practice Extension Set II Question 1.
A.9C	3, 4, 5	**To support students:** • Use Skills Practice Sets I.B, I.C, and II.B for additional practice. • Review Lesson 2 Assignment Practice Questions 1 and 3. **To challenge students:** • Extend student knowledge with Skills Practice Extension Set II Question 2.
A.9D	1, 2, 7	**To support students:** • Use Skills Practice Sets II.A and III.D for additional practice. • Review Lesson 2 Assignment Practice Question 3.
A.9E	6	**To support students:** • Use Skills Practice Set III.A for additional practice. • Review Lesson 3 Assignment Practice Question 1. **To challenge students:** • Extend student knowledge with Skills Practice Extension Set III.
A.11B	12	**To support students:** • Use Skills Practice Spaced Practice Set II for additional practice.
A.12B	9	**To support students:** • Use Skills Practice Set III.C for additional practice. • Review Lesson 3 Assignment Practice Question 1.
NOTE: Both teachers and administrators should refer to the Assessment Guidance and Analysis section of the Course and Implementation Guide for additional support in analyzing and responding to student data.		

*Bold TEKS = Readiness Standard